

Principles of Communications

ECS 332

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7. Pulse Modulation, ISI, and Pulse Shaping



Office Hours:

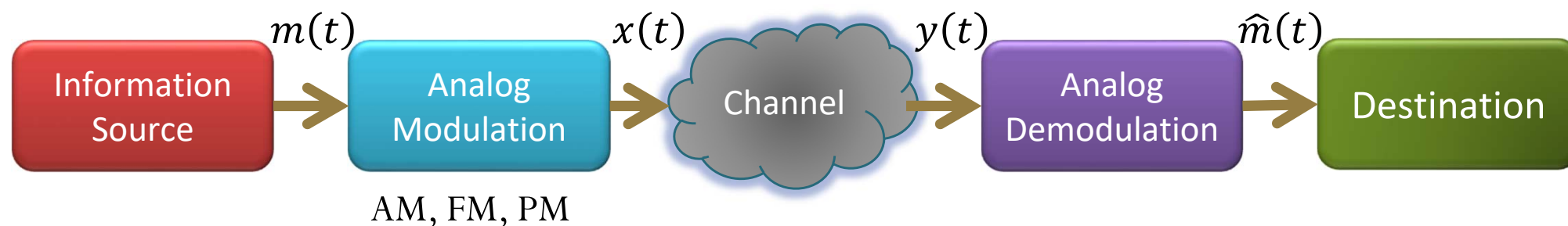
BKD, 6th floor of Sirindhralai building

Wednesday 14:30-15:30

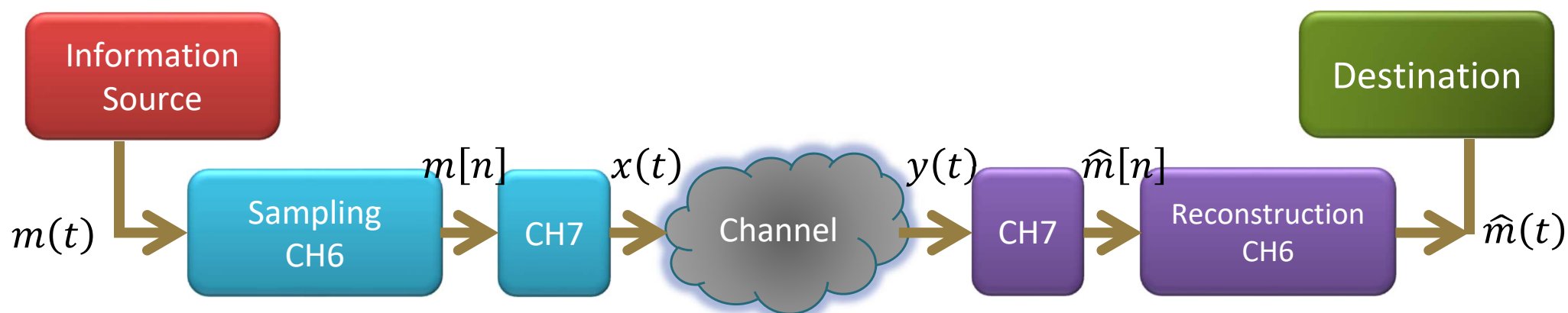
Friday 14:30-15:30

CH 3-5 vs. CH 6-7

- Chapters 3-5



- Chapters 6-7



Chapter 7

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- Goal: Convert $m[n]$ into an analog signal $x(t)$ that
 - has no inter-symbol interference
 - uses small bandwidth.



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7.1 Analog Pulse Modulation

ASCII in MATLAB

```
>> str='I love SIIT';      text string
>> real(str)
```

```
ans =      (decimal) ASCII representation of the text string
      73      32     108     111     118     101      32      83      73      73      84
```

```
>> dec2base(str,2)
```

```
ans =
1001001
0100000
1101100
1101111
1110110
1100101
0100000
1010011
1001001
1001001
1010100
```

binary (base 2)
representation of the
decimal numbers

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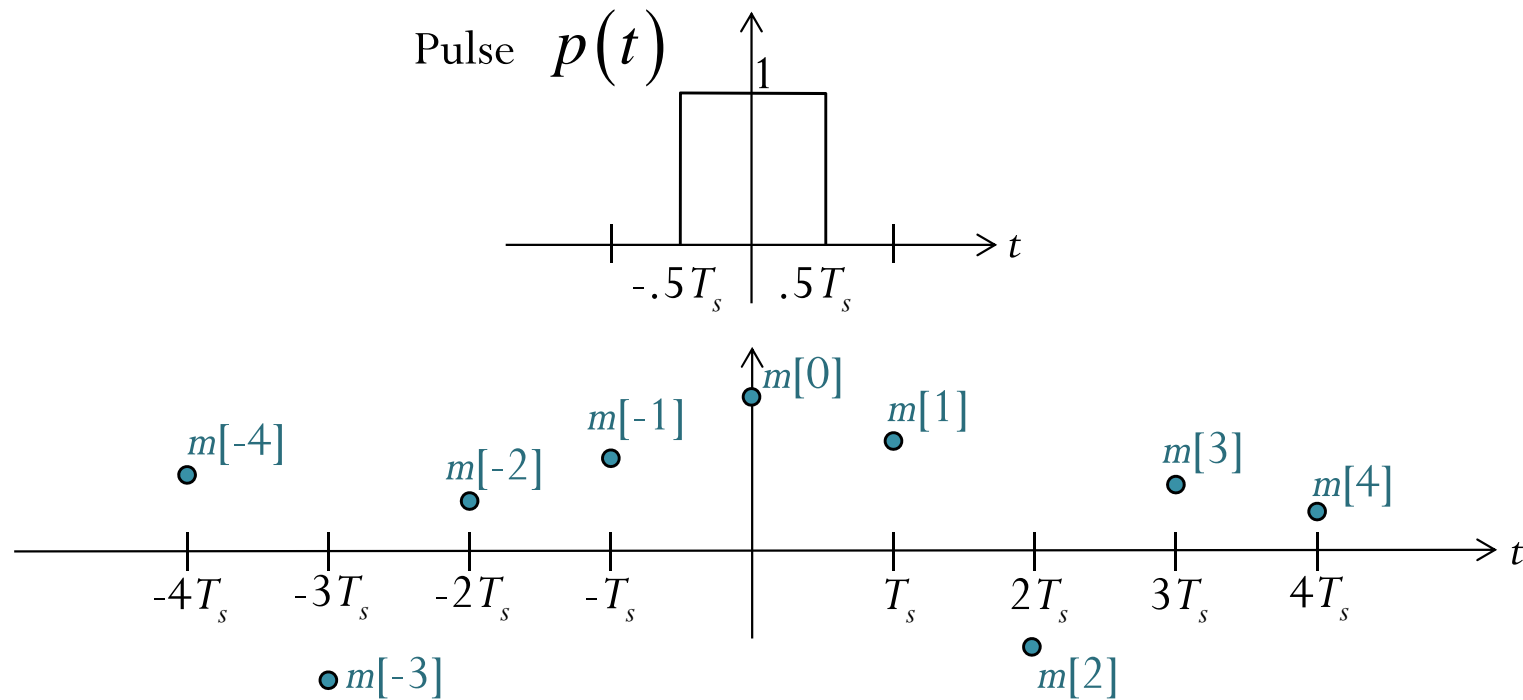
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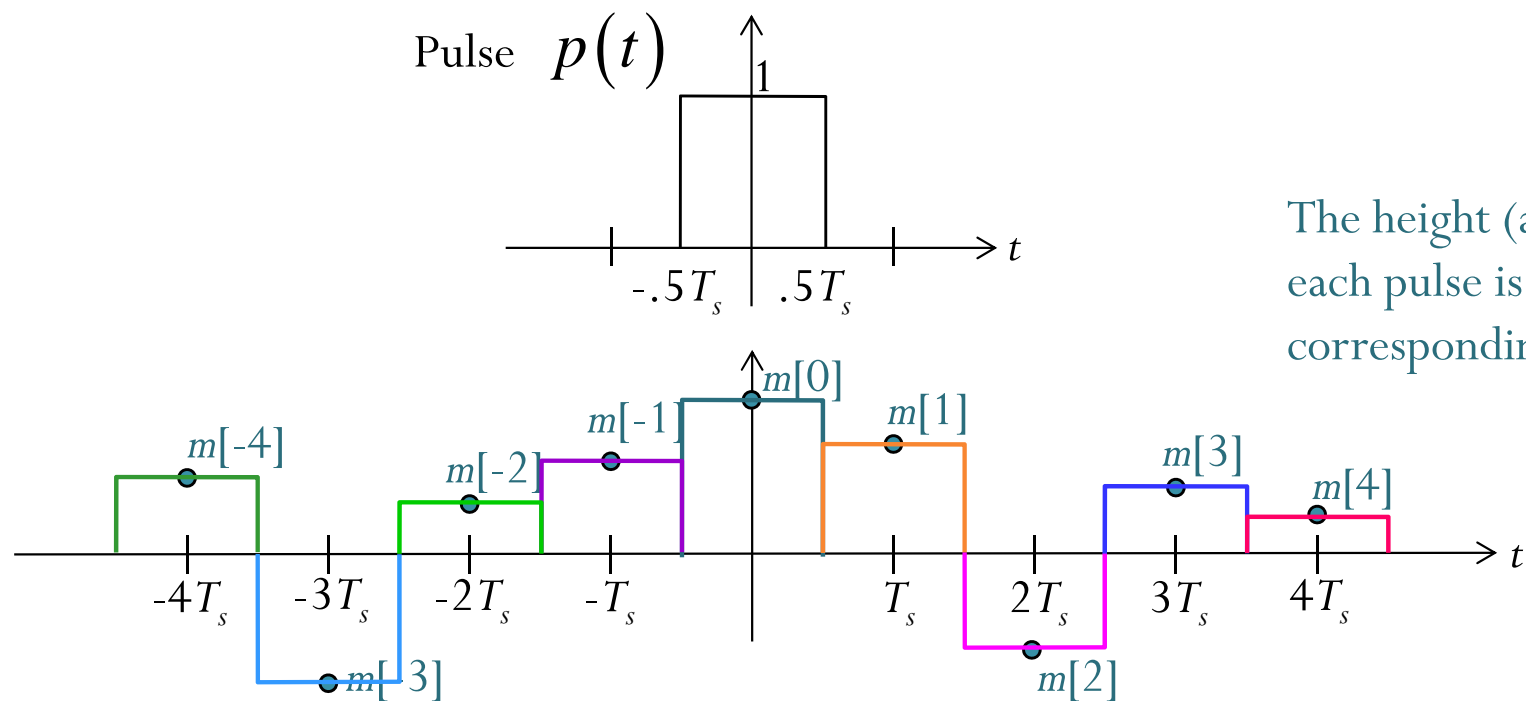
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7.2 PAM

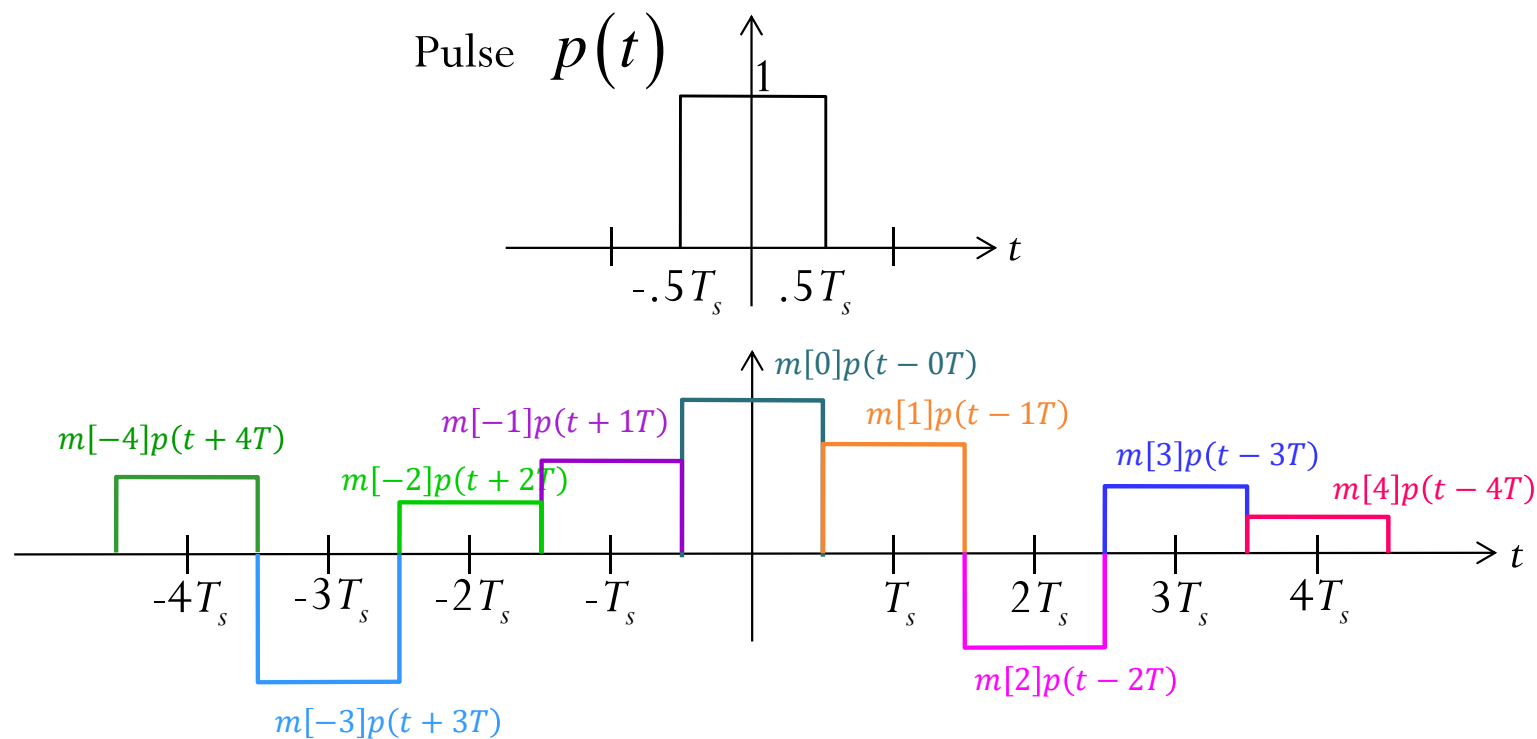
PAM: Example 7.11 (Figure 57)



PAM: Example 7.11 (Figure 57)



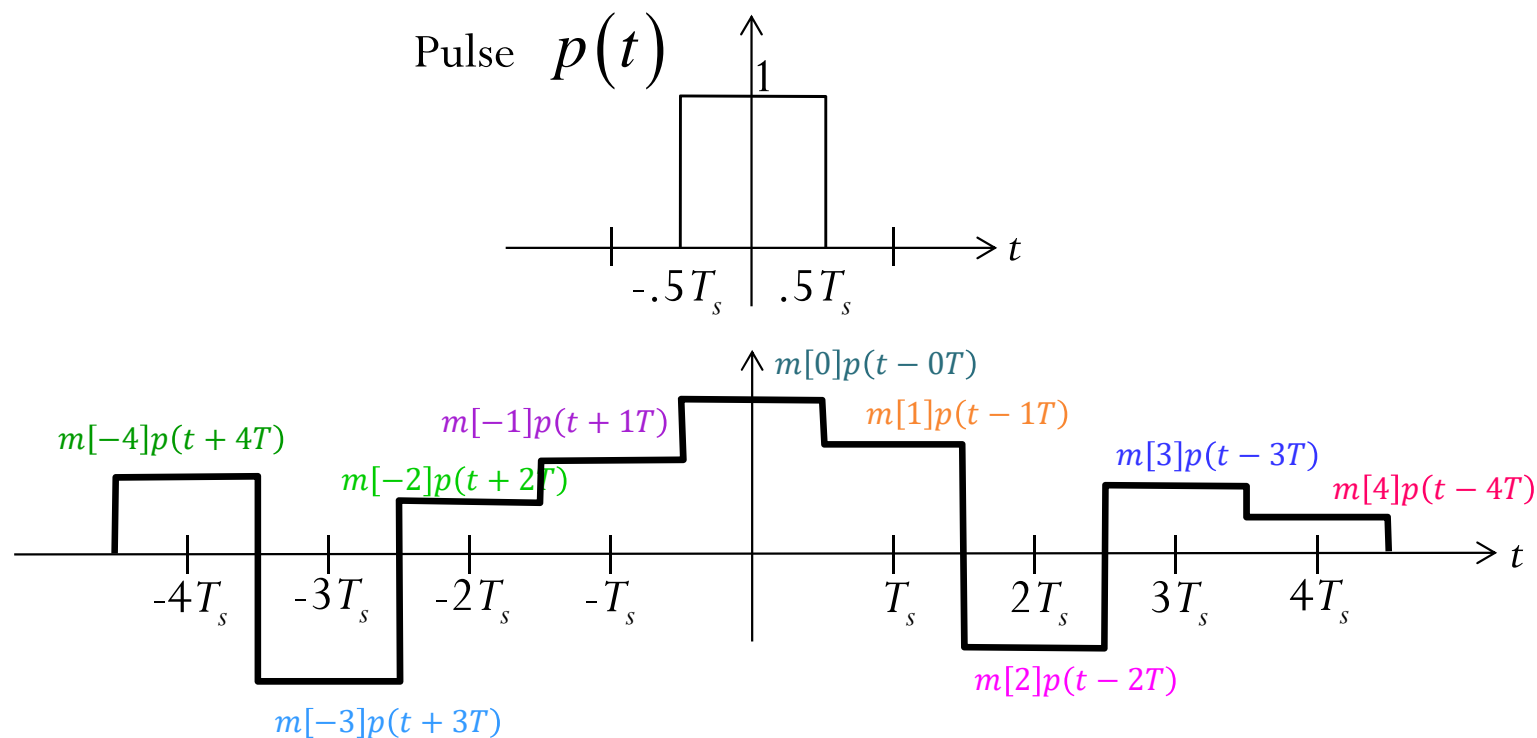
PAM: Example 7.11 (Figure 57)



$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n] p(t - nT)$$



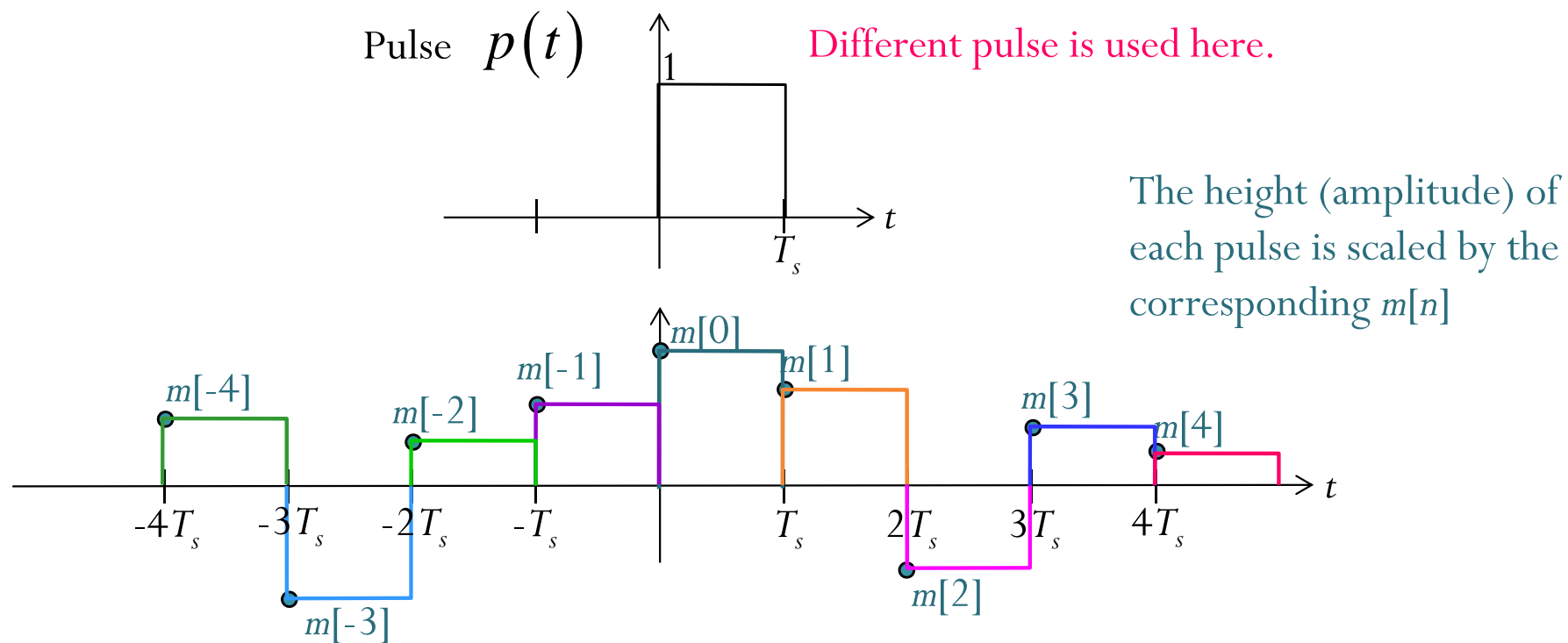
PAM: Example 7.11 (Figure 57)



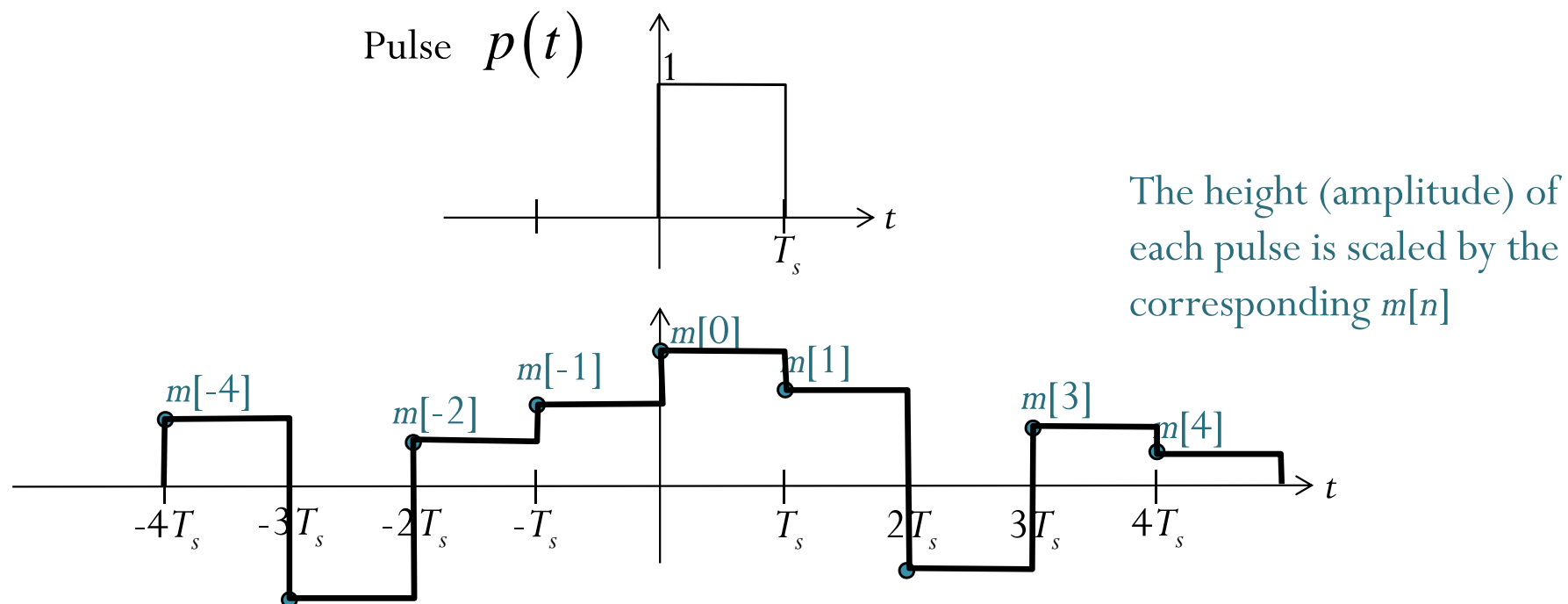
$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n] p(t - nT)$$



PAM: Example 7.12 (Figure 58)



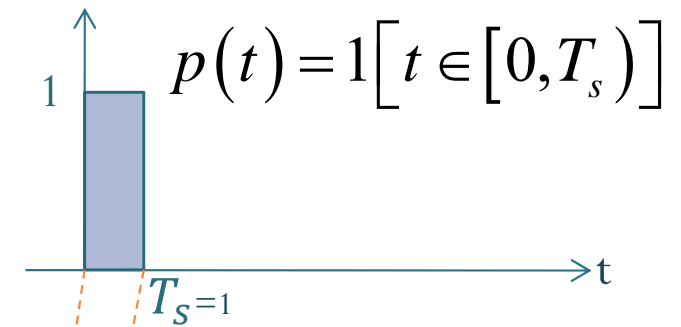
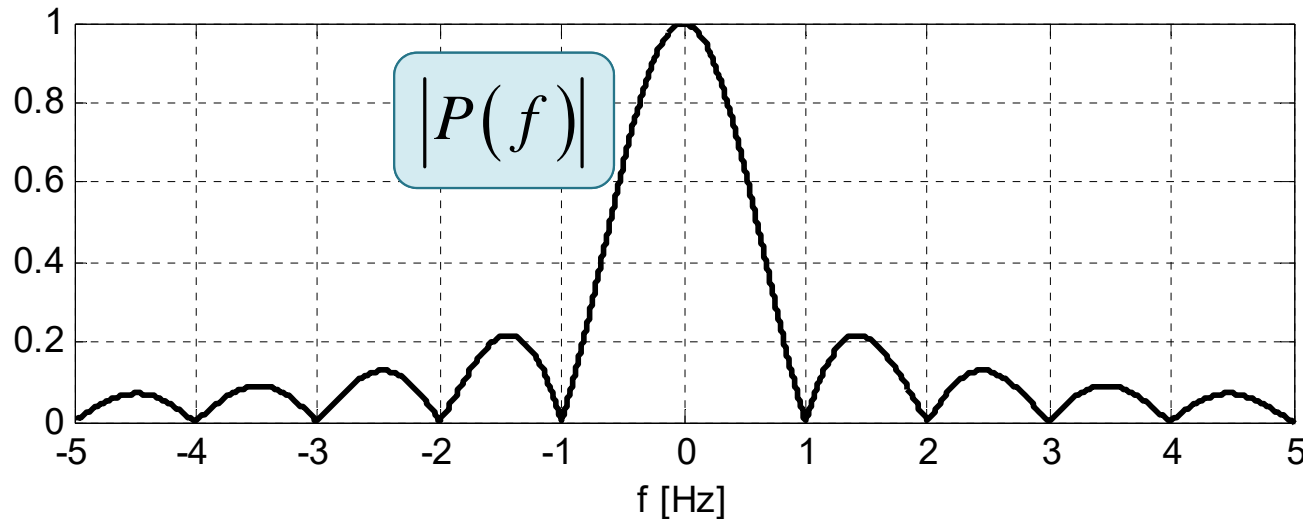
PAM: Example 7.12 (Figure 58)



$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n] p(t - nT)$$



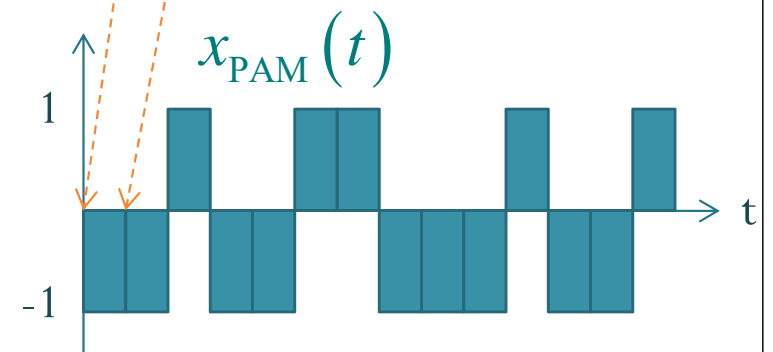
Review: Spectrum of PAM signal



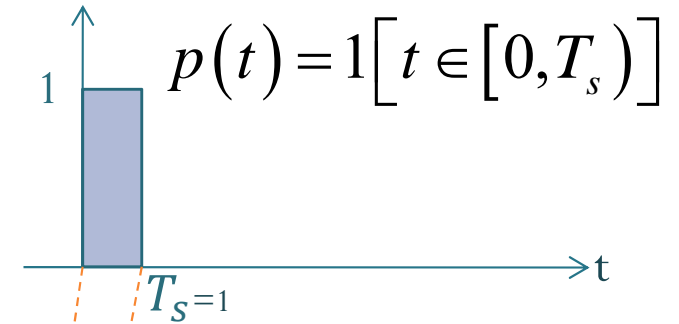
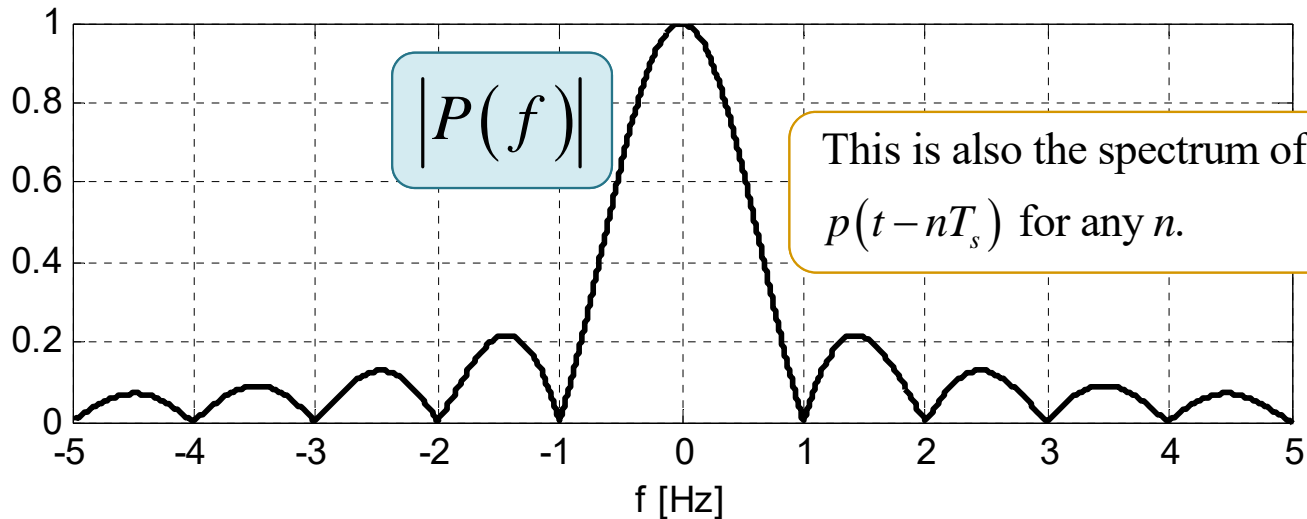
$$\mathbf{m} = [-1, -1, 1, -1, -1, 1, 1, -1, -1, -1, 1, -1, -1, 1, 1, -1, -1, -1, -1, 1, -1, -1, 1, -1, 1]$$

$$x_{\text{PAM}}(t) = \sum_n m[n] p(t - nT_s)$$

Can you sketch the spectrum of $x_{\text{PAM}}(t)$?



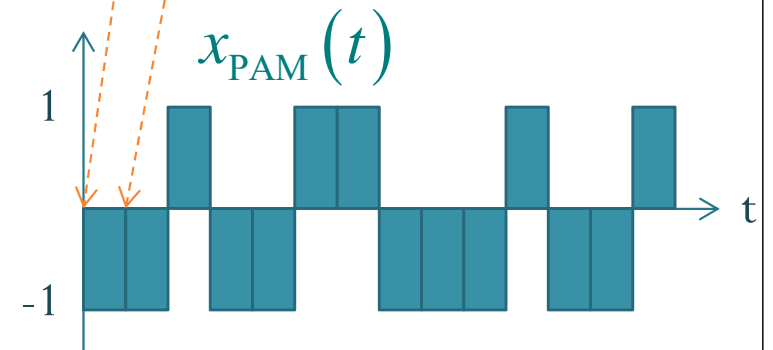
$X_{\text{PAM}}(f)$ (2/4)



$$\mathbf{m} = [-1, -1, 1, -1, -1, 1, 1, -1, -1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, -1, -1, 1, -1, 1, -1, 1]$$

$$x_{\text{PAM}}(t) = \sum_n m[n] p(t - nT_s)$$

Does this mean $|X_{\text{PAM}}(f)|$ will simply be a sum of $|P(f)|$ and therefore its shape will be similar to $|P(f)|$?



Important Properties of \mathcal{F}

$$\{x * y\}(t) = \int_{-\infty}^{\infty} x(\mu)y(t - \mu)d\mu = \int_{-\infty}^{\infty} x(t - \mu)y(\mu)d\mu$$

Convolution Properties:

$$x * y \xrightleftharpoons{\mathcal{F}} X \times Y$$

$$x \times y \xrightleftharpoons{\mathcal{F}} X * Y$$

Note that the magnitude of this is simply $|G(f)|$

Shifting Properties:

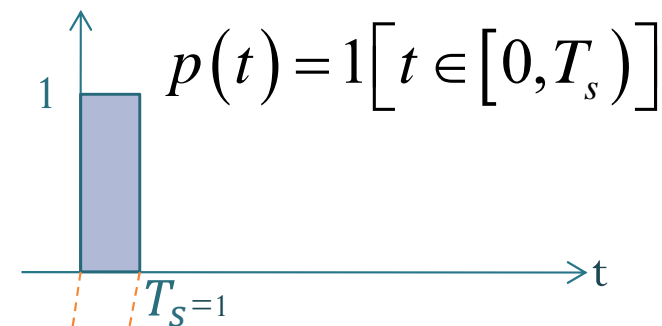
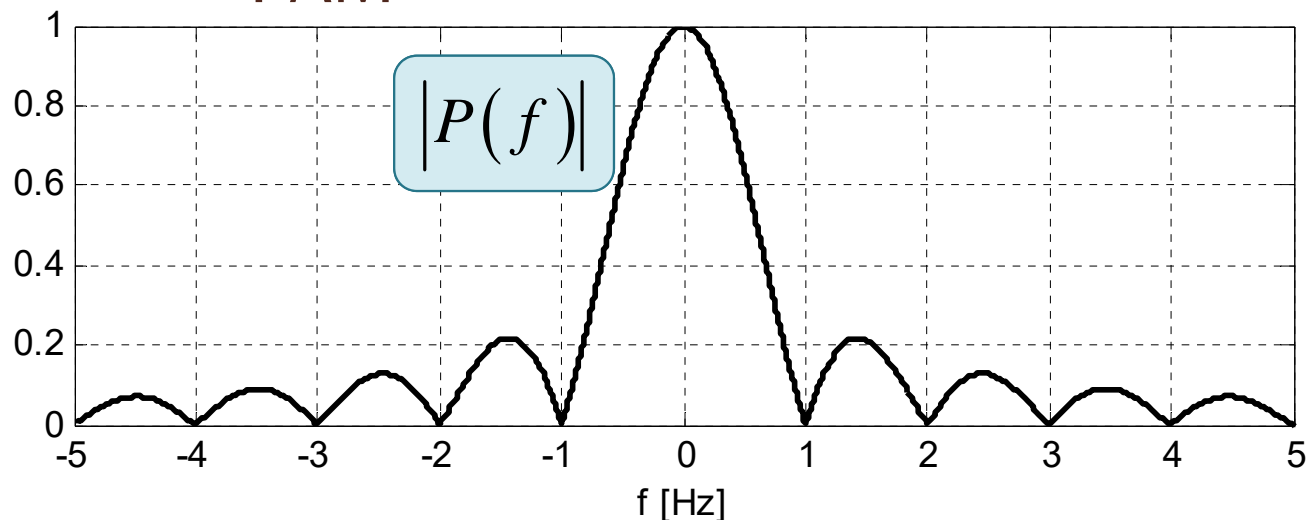
$$g(t - t_0) \xrightleftharpoons{\mathcal{F}} e^{-j2\pi ft_0} G(f)$$

$$e^{j2\pi f_0 t} g(t) \xrightleftharpoons{\mathcal{F}} G(f - f_0)$$

Modulation:

$$g(t) \cos(2\pi f_c t) \xrightleftharpoons{\mathcal{F}} \frac{1}{2} G(f - f_c) + \frac{1}{2} G(f + f_c)$$

$X_{\text{PAM}}(f)$ (3/4)



$$\mathbf{m} = [-1, -1, 1, -1, -1, 1, 1, -1, -1, -1, 1, -1, -1, 1, 1, -1, -1, -1, -1, 1, -1, -1, 1]$$

$$x_{\text{PAM}}(t) = \sum_n m[n] p(t - nT_s)$$

$$\xrightarrow{\mathcal{F}} X_{\text{PAM}}(f) = \sum_n m[n] P(f) e^{-j2\pi fnT_s}$$

$$= P(f) \sum_n m[n] e^{-j2\pi fnT_s}$$

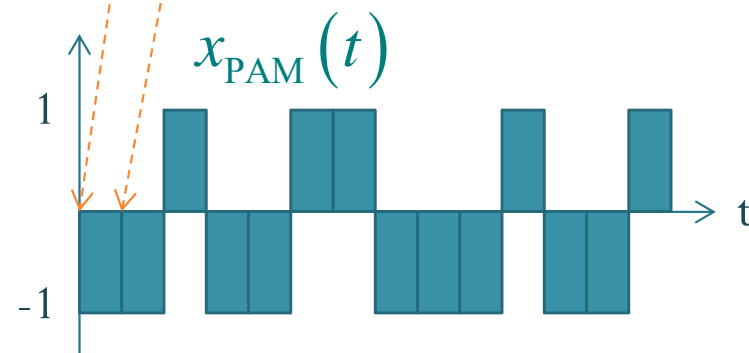
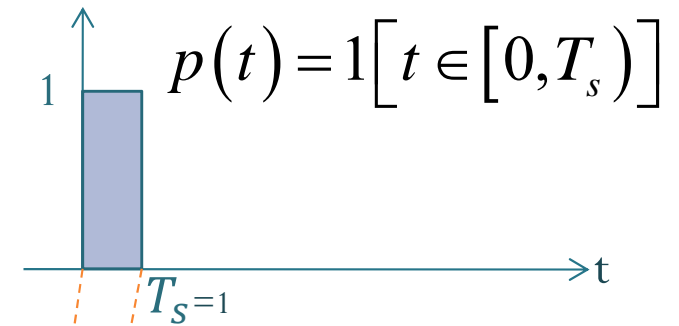
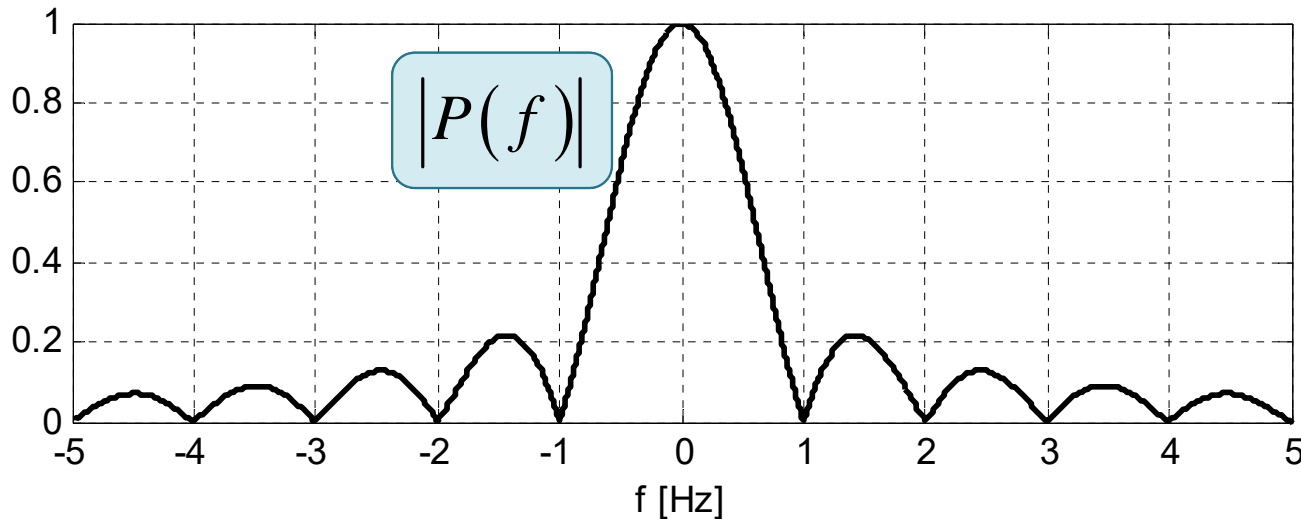
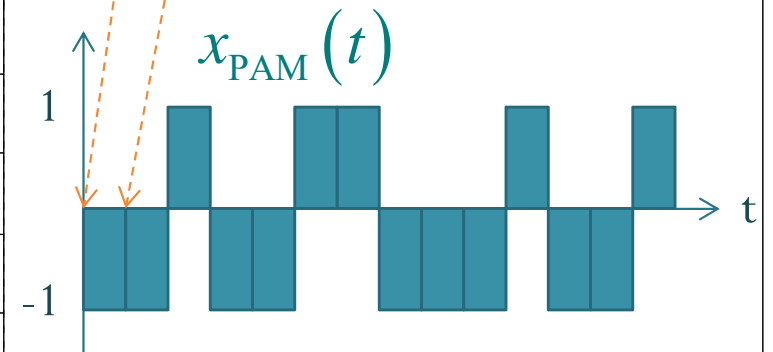
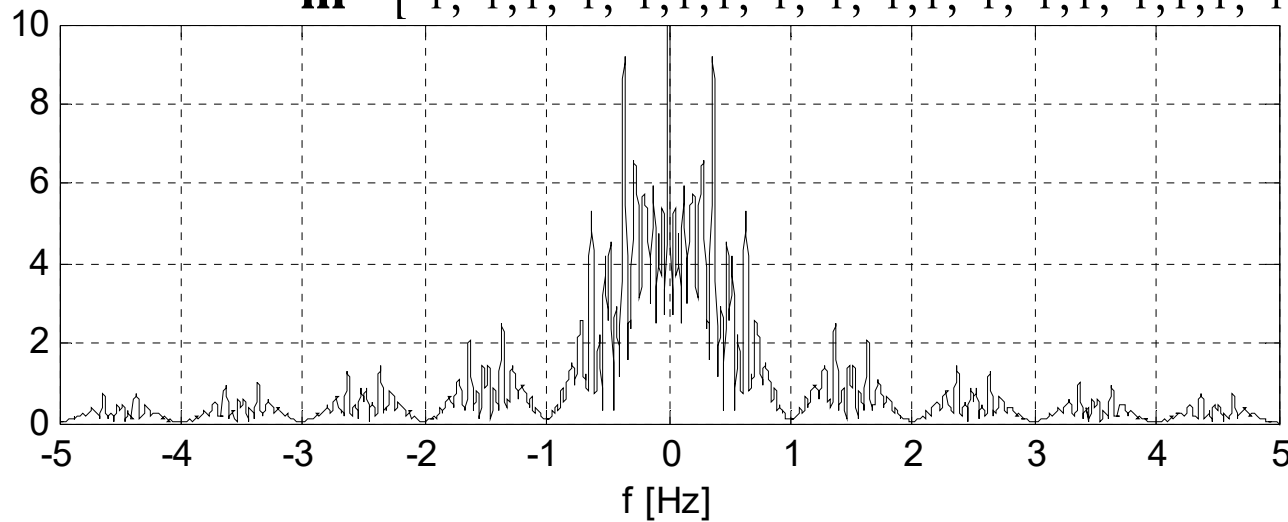


Figure 60

$X_{\text{PAM}}(f)$ (4/4)



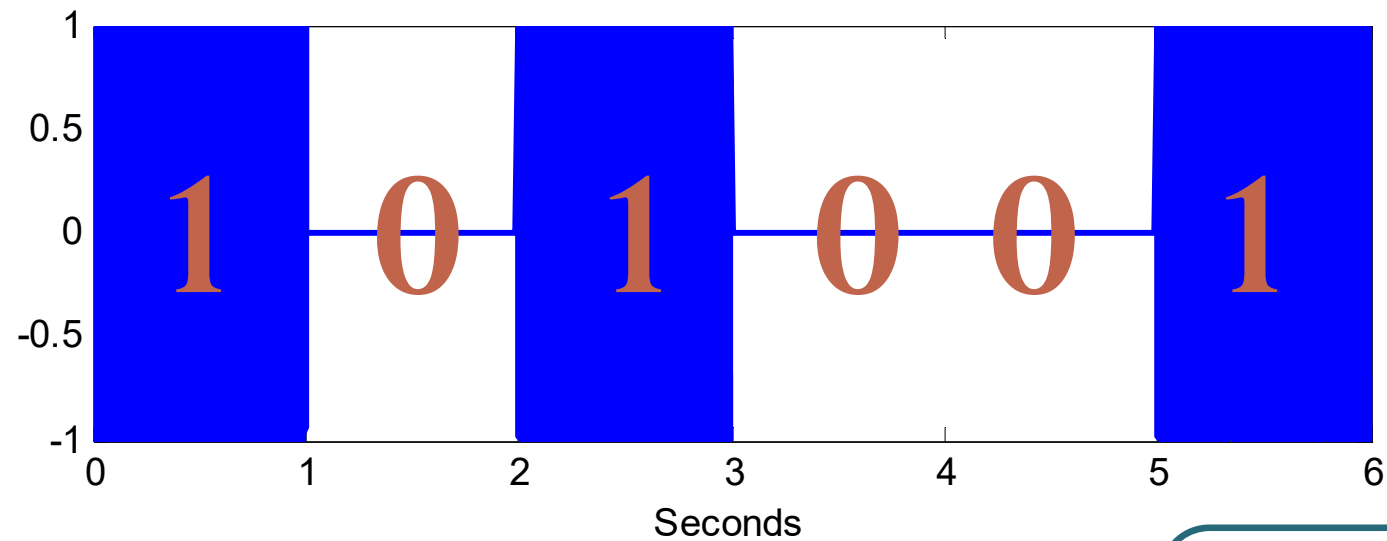
$\mathbf{m} = [-1, -1, 1, -1, -1, 1, 1, -1, -1, -1, 1, -1, -1, 1, 1, -1, -1, -1, -1, -1, -1, 1, -1, 1]$



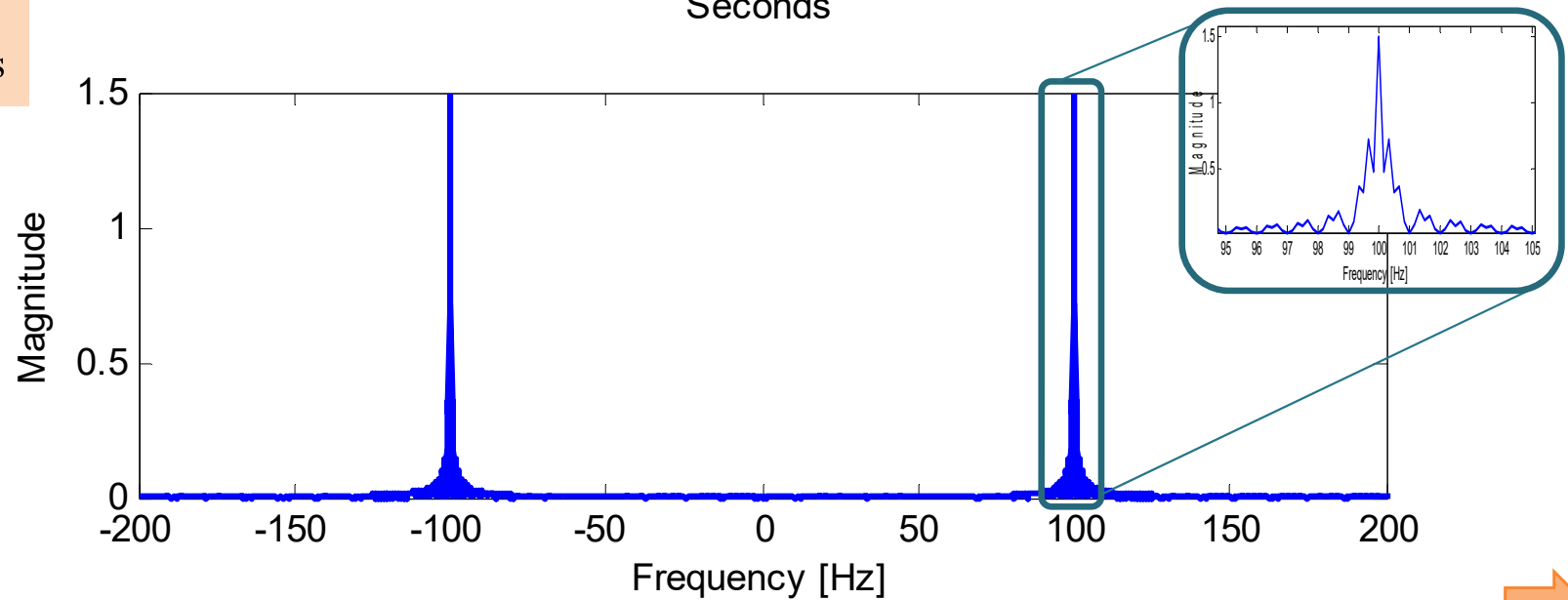
$$x_{\text{PAM}}(t) = \sum_n m[n] p(t - nT_s) \xrightarrow{\mathcal{F}} X_{\text{PAM}}(f) = P(f) \sum_n m[n] e^{-j2\pi fnT_s}$$



A revisit to an earlier OOK Example

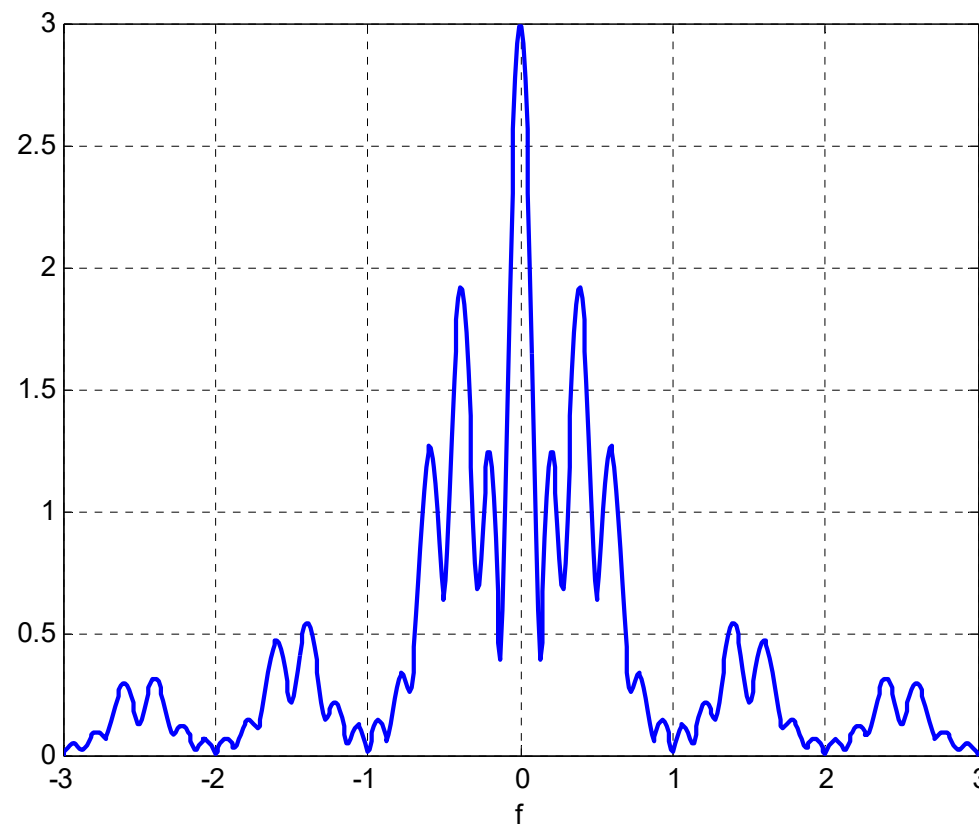
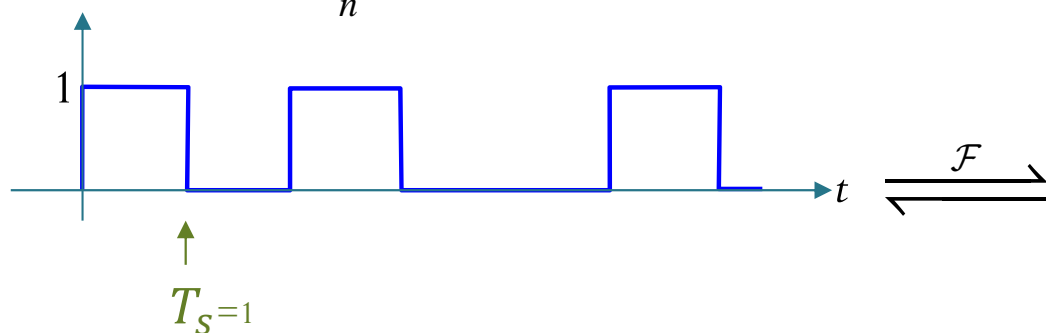


$f_c = 100$ Hz
Bit rate = 1 bps



A revisit to an earlier OOK Example

$$x_{\text{PAM}}(t) = \sum_n m[n] p(t - nT_s)$$



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7.3 ISI and Pulse Shaping

Section 7.3

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- Original Goal: Convert $m[n]$ into an analog signal $x(t)$ that
 - has no inter-symbol interference
 - uses small bandwidth.

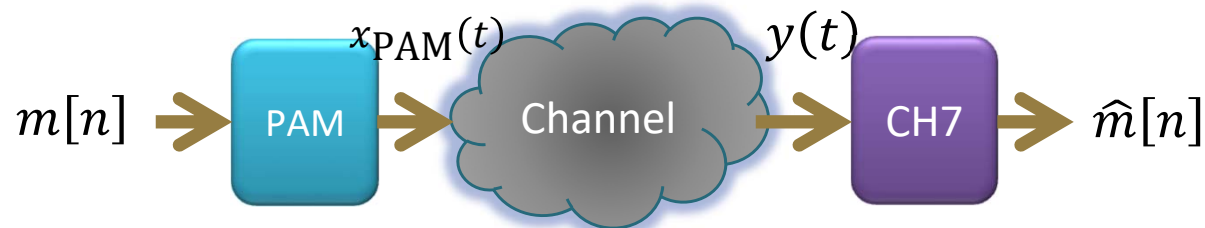


Section 7.3

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- In Section 7.2, we choose to convert $m[n]$ into an analog signal by using PAM:

$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]p(t - nT_s)$$

- Current Goal: Choose the pulse shape $p(t)$ so that $x(t)$
 - has no inter-symbol interference
 - uses small bandwidth.

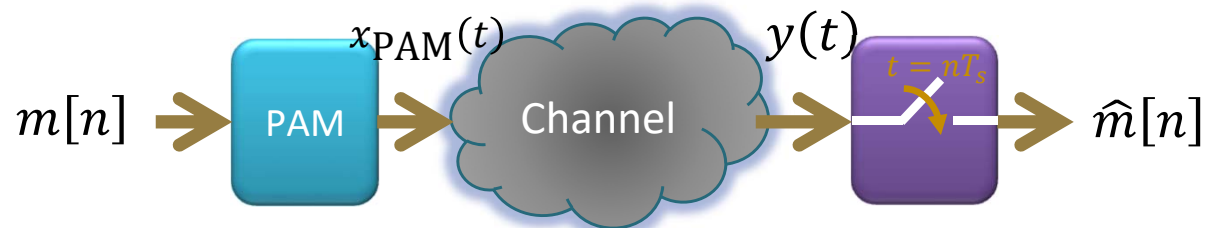


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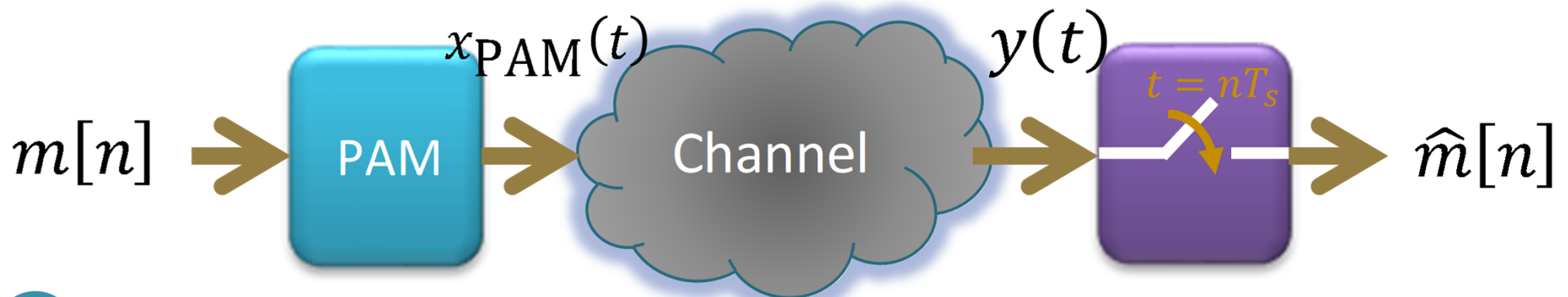


Section 7.3: Conditions for good pulse

- Start with a discrete-time signal (a sequence of numbers $m[n]$).
- In Section 7.2, we choose to convert $m[n]$ into an analog signal by using PAM:

$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]p(t - nT_s)$$

- Current Goal: Choose the pulse shape $p(t)$ so that $x(t)$
 - has no inter-symbol interference: $\hat{m}(n) = m(n)$ for all n
 - uses small bandwidth.

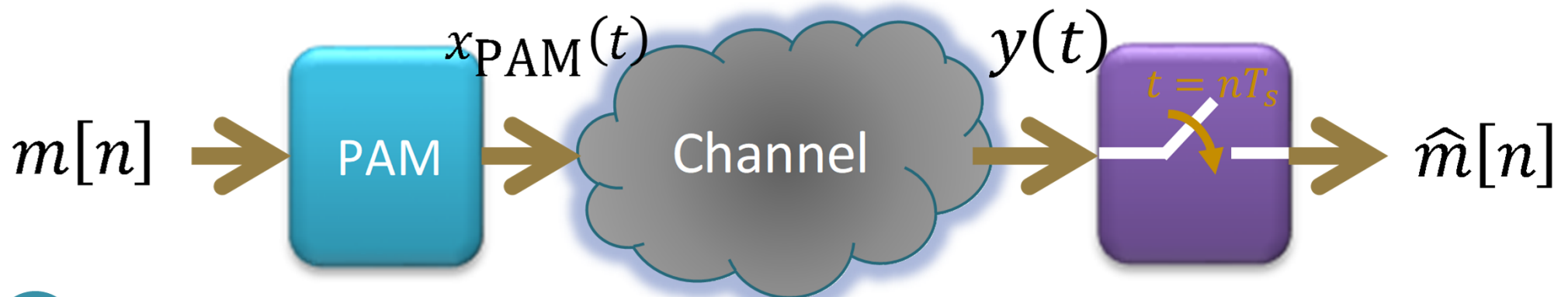


Conditions for good pulse

[7.17] Two conditions:

(a1) $\hat{m}(n) = m(n)$ for all n .

(b) $P(f)$ is band-limited (with small BW).



Conditions for good pulse

[7.17] Two conditions:

$$(a2) \quad p(t) = \begin{cases} 1, & t = 0, \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

(b) $P(f)$ is band-limited (with small BW).

These two conditions are in two different domains.



Nyquist's (first) Criterion for Zero ISI

- Two equivalent definitions for **Nyquist pulse**:
- In the **time domain**,

(a2)

$$p(t) = \begin{cases} 1, & t = 0, \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

$\frac{1}{T}$ = **signaling rate**
measured in
[symbols per second]
or
[baud]

Symbol “duration”
“interval”

- In the **frequency domain**,

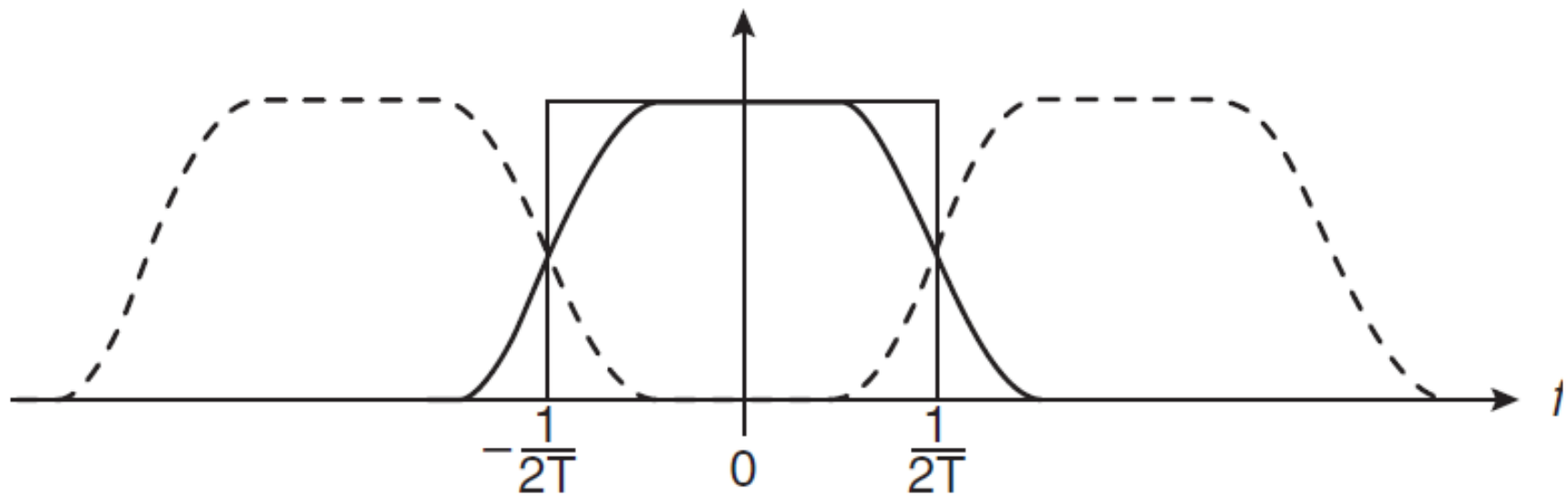
(a3)

$$\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T, \quad |f| \leq \frac{1}{2T}$$

The sum on the left is always periodic with period $\frac{1}{T}$. So, it is sufficient to check the criterion only on this interval.



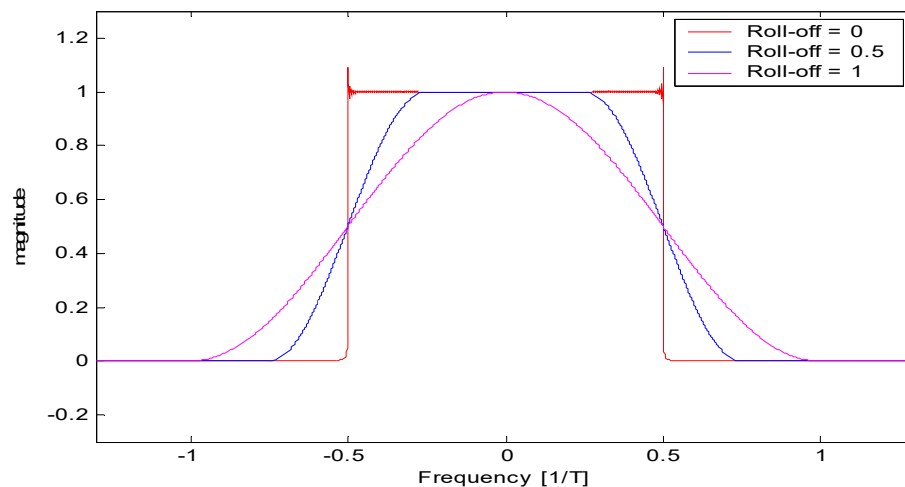
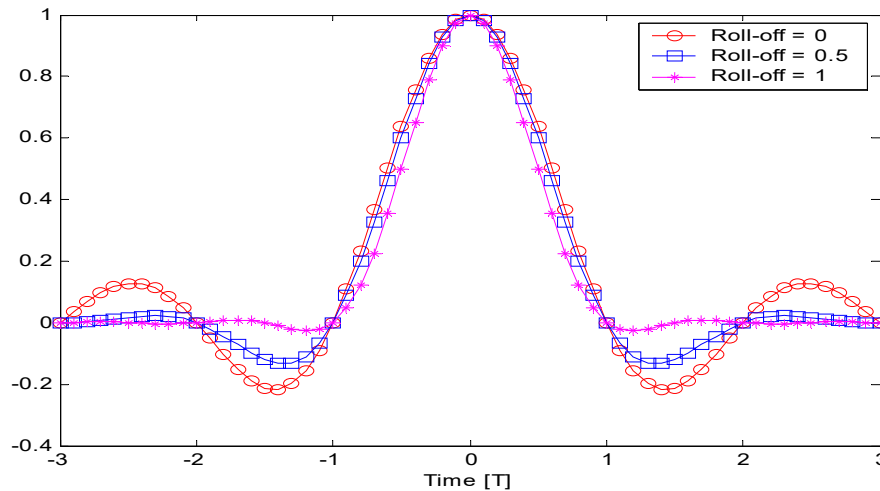
Nyquist criterion



[Blahut, 2008, Fig 2.9]



Raised Cosine Pulses



For fixed nonzero α , the tails decay as $1/t^3$ for large $|t|$.

Although the pulse tails persist for an infinite time, they are eventually small enough so they can be truncated with only negligible perturbations of the zero crossings.

$$p_{RC}(t; \alpha) = \frac{\cos \frac{\alpha \pi t}{T}}{1 - \frac{4\alpha^2 t^2}{T^2}} \operatorname{sinc} \frac{\pi t}{T}$$

$$= \frac{\cos \frac{\alpha \pi t}{T}}{1 - \frac{4\alpha^2 t^2}{T^2}} \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}}$$